6. (a) Evaluate : **6**

$$\int_0^\infty e^{-2t} t \sin 3t \, dt .$$

(b) Solve the differential equation

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t, \; y(0) = 2$$ and

$y'(0) = -1$. **9**

Unit IV

7. (a) Find the Fourier Transform of : **8**

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3} \right) \cos \frac{x}{2} \, dx .$$

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B.Tech. EXAMINATION, 2024

(Third Semester)

(C Scheme) (Main & Re-appear)

(CSE)

MATHEMATICS-III

MATH207C

Partial Differential Equations and Transforms

Time : 3 *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions selecting at least *one* question from each Unit. All questions carry equal marks.

Shiksha Sanchar

Unit I

1. (a) Form a partial differential equation by eliminating the functions f and g from $y = f(x - at) + g(x + at)$. **7**

(b) Solve : **8**

$$px + qy = pq.$$

2. (a) Solve : **7**

$$x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2).$$

(b) Solve : **8**

$$4\frac{\partial^2 z}{\partial x^2} + 12\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = e^{3x-2y}$$

Unit II

3. (a) Classify the partial differential equation : **6**

$$(1 - x^2)u_{xx} - 2xy\ u_{xy} + (1 - y^2)u_{yy} + x\ u_x + 3x^2y\ u_y - 2u = 0.$$

(b) Using, the method of separation of variables, solve : **9**

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6e^{-3x}$.

4. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, in a rectangle in the xy-plane with $u(x, 0) = 0$, $u(x, b) = 0$, $u(0, y)$ and $u(a, y) = f(y)$ parallel to y axis. **15**

Unit III

5. (a) State and Prove the orthogonal Property of Chebyshev Polynomials. **8**

(b) Apply the convolution theorem to find

$$L^{-1}\left\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right\}.$$ **7**

Shiksha Sanchar

(b) If F(s) is the complex Fourier Transform of $f(x)$, then show that : **7**

$$F[f(x) \cos ax] = \frac{1}{2} [F(s + a) + F(s - a)].$$

8. (a) Find the Z-transform of $c^k \cos \alpha k$, $k \geq 0$. **8**

(b) Find the inverse Z-transform of $\frac{1}{(z-5)^3}, |z| > 5$. **7**

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